

High-fidelity quantum memory using nitrogen-vacancy center ensemble for hybrid quantum computation

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We study a hybrid quantum computing system using nitrogen-vacancy center ensemble (NVE) as quantum memory, current-biased Josephson junction (CBJJ) superconducting qubit fabricated in a transmission line resonator (TLR) as quantum computing processor and the microwave photons in TLR as quantum data bus. The storage process is seriously treated by considering all kinds of decoherence mechanisms. Such a hybrid quantum device can also be used to create multi-qubit W states of NVEs through a common CBJJ. The experimental feasibility and challenge are justified using currently available technology.

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With experimental progress in fabrication of solid-state systems and in manipulation of qubits in atomic, molecular and optical systems, a lot of efforts have been given to explore the possibility of building hybrid quantum devices [1]. Combining the advantages of component systems in a compatible experimental setup, hybrid quantum device may allow us to focus on the quantum coherent interface between different kinds of qubits. Examples include storage of quantum information from fragile qubits, e.g., superconducting qubit [2], to long-lived quantum memories such as ultracold ⁸⁷Rb atomic ensemble [3] and polar molecular ensemble [4, 5]. To achieve appreciable coupling strengths, these systems often resort to large electric-dipole interactions [3, 4], e.g., employing optically excited Rydberg states [3].

However, magnetic-dipole interactions may be more desirable due to sufficiently long coherence times achieved in systems with spin states storing quantum information [6]. Besides, electronic spin degrees of freedom may provide excellent quantum memory owing to their weak magnetic interaction with the environment. For example, the nitrogen-vacancy (NV) centers in diamond have significant electronic spin lifetime, narrow-band optical transitions, as well as the possibility of coherent manipulation at room temperature [7, 8]. Several recent highlights include two parallel experiments demonstrating strong magnetic couplings (~ 10 MHz) of a transmission line resonator (TLR) to an ensemble of $10^{11} \sim 10^{13}$ NV centers [9], and to *N* substitution (P1) centers [10], where the strong spin-field coupling benefits from the en-

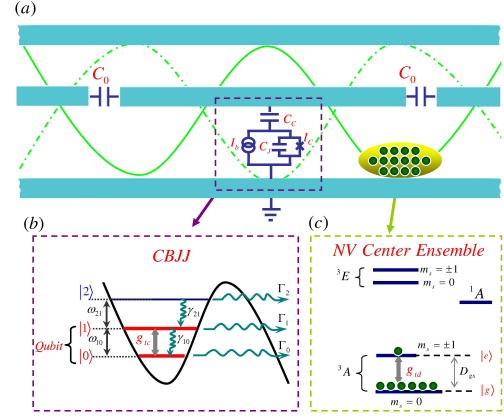


FIG. 1: (Color online) (a) A NVE and a CBJJ are coupled via a common quantized field of the TLR acting as a quantum data bus. The CBJJ is fabricated at the antinodes of the electric field of the full-wave mode of the TLR. (b) Level scheme of a CBJJ qubit. When the bias current I_b is driven close to the critical bias I_c , there exist only a few bound states $|n\rangle_C$ with energy E_n in each washboard well [12]. (c) Level structure of a NVE where the electronic ground and first excited states are electron spin triplet states ($S=1$), and $D_{gs}/2\pi = 2.87$ GHz is the zero-field splitting between the lowest energy $m_s = 0$ sublevel and the $m_s = \pm 1$ sublevels.

hancement of the coupling strength by a factor \sqrt{N} for an ensemble consisting of N qubits. It implies the possibility of using electronic spin ensemble to construct a good quantum memory for superconducting qubits.

In this work, we study a hybrid quantum device which uses nitrogen-vacancy center ensemble (NVE) as a spin-based quantum memory and uses current-biased Josephson junction (CBJJ) phase qubit [11–15] fabricated in a

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TLR [16] as a quantum computing processor, which implements rapid quantum logic gates. The spin qubits are coupled to quantized modes of TLR through an effectively enhanced magnetic-dipole interaction, which enables high-quality storage of arbitrary quantum states of a CBJJ. Considering decoherence in experimentally available systems, we show the feasibility to achieve a high-fidelity quantum memory with either resonant interaction (RI) or dispersive interaction (DI) by modulating the external parameters of the CBJJ to independently tune both the CBJJ's transition frequency and CBJJ-TLR coupling strength. We also discuss a potential application of such a hybrid quantum device for preparing multi-qubit W states of NVEs.

As illustrated in Fig. 1(a), the hybrid quantum device we study is the CBJJ-TLR-NVE system governed by $H_{tot} = H_C + H_T + H_D + H_{TC} + H_{TD}$. The employed CBJJ is designed using Josephson junctions to make low loss nonlinear oscillator so that one of the key elements of the CBJJ, i.e., sufficiently large anharmonicity, could be obtained for preventing qubit operations from exciting other transitions in these energy levels. So the Hamiltonian of CBJJ can be simply modeled as $H_C = \frac{\hbar}{2}\omega_{10}\sigma^z$, where $\sigma^z = |1\rangle_C\langle 1| - |0\rangle_C\langle 0|$ is the Pauli spin operator, and $\hbar\omega_{10}$ is the lowest energy-level spacing of the CBJJ. With the energy of $|0\rangle_C$ to be energy zero point, the energy-level spacings can be expressed as $\omega_{10} \simeq 0.9\omega_p$ and $\omega_{21} \simeq 0.81\omega_p$ with $\omega_p = \sqrt[4]{(2 - 2I_b/I_c)(2\pi I_c/\Phi_0 C_J)^2}$ the plasma oscillation frequency at the bottom of the well [12], the bias current I_b , the critical current I_c , the junction capacitance C_J , and the flux quantum $\Phi_0 = h/2e$.

The Hamiltonian of the microwave-driven TLR (with length L , inductance F_t , capacitance C_t , wiring capacitors C_0 , and coupling capacitors C_c) is $H_T = \hbar\omega_c a^\dagger a$ where a (a^\dagger) is the annihilation (creation) operator of the full-wave mode, and the frequency of this mode is slightly renormalized by the wiring and coupling capacitor as $\omega_c \approx 2\pi(1 - \varepsilon_1 - \varepsilon_2)/(\sqrt{F_t C_t})$ with $\varepsilon_1 = 2C_0/C_t$ and $\varepsilon_2 = C_c/C_t$. The voltage and current distribute inside the TLR as

$$\begin{aligned} V_{tlr}(x) &= \sqrt{\hbar\omega_c/C_t}(a^\dagger + a)\cos(kx + \delta_0), \\ I_{tlr}(x) &= -i\sqrt{\hbar\omega_c/F_t}(a - a^\dagger)\sin(kx + \delta_0), \end{aligned} \quad (1)$$

where $k = 2\pi/L$, and the small phase δ_0 meets the condition $\tan \delta_0 = 2\pi\varepsilon_2$. The TLR can be capacitively coupled to the CBJJ by the coupling capacitors C_c and junction capacitance C_J . This resonator acts as the channel to control, couple, and read out the states of the qubit [17]. In this circuit quantum electrodynamic architecture, the CBJJ-TLR interaction Hamiltonian can be written as $H_{TC} = \hbar g_{tc}(\sigma^+ a + \sigma^- a^\dagger)$ with $\sigma^+ = |1\rangle_C\langle 0|$ ($\sigma^- = |0\rangle_C\langle 1|$) the CBJJ's rising (lowering) operator. $g_{tc} = [2C_t(C_J + C_c)]^{-1/2}\omega_c C_c \cos \delta_0$ is a tunable qubit-resonator coupling strength [14] due to the flexibility of CBJJ-qubits, allowing access to a wide range of tunable experimental parameters for I_b and C_J . A similar system

was presented in Ref. [15] for the realization of entanglement between two distant NVEs.

The Hamiltonian of a NVE reads $H_D = \frac{\hbar}{2}\omega_{eg}S^z$, where S^l ($l = z, \pm$) $= (1/\sqrt{N})\sum_i \tau_i^l$ is the collective spin operator for the NVE with $\tau^z = |e\rangle\langle e| - |g\rangle\langle g|$, $\tau^+ = |e\rangle\langle g|$ and $\tau^- = |g\rangle\langle e|$. The operator S^+ can create symmetric Dicke excitation states $|n\rangle_D$, among which the NVE could be defined in the lowest two states $|0\rangle_D = |g_1 g_2 \dots g_N\rangle$ and $|1\rangle_D = S^+ |0\rangle_D = (1/\sqrt{N})\sum_{k=1}^N |g_1 \dots e_k \dots g_N\rangle$. All the spins in NVE interact symmetrically with a single mode of electromagnetic field because the mode wavelength is larger than the spatial dimension of the NVE if the spin ensemble is placed near the TLR's field antinode. So the NVE can be coupled to the TLR by magnetic-dipole coupling with the corresponding Hamiltonian $H_{TD} = \hbar g_{td}(S^+ a + S^- a^\dagger)$, which is a Jaynes-Cummings-type interaction with $g_{td} = \sqrt{N}g_s$, and g_s being the single NV vacuum Rabi frequency.

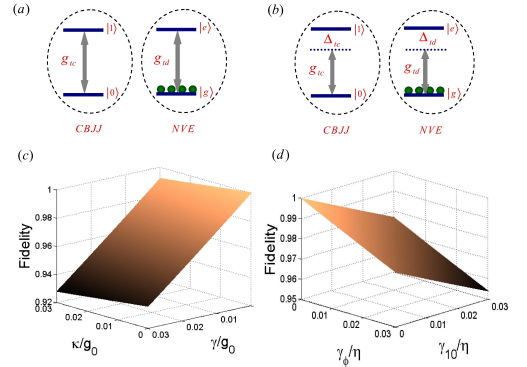


FIG. 2: (Color online) (a) Level Scheme of the CBJJ and NVE coupled to the TLR in resonant case. (b) Level Scheme of the CBJJ and NVE coupled to the TLR in dispersive case. (c) The fidelity of the quantum state transfer versus the dimensionless TLR decay rate κ/g_0 and CBJJ decay rate γ/g_0 in the case of $\alpha = \beta = 1/\sqrt{2}$, where we have set $\gamma = \gamma_\phi = \gamma_{10} = \Gamma_1$, and $t_R = \pi/\sqrt{2}g_0$. (d) The fidelity of the quantum state transfer versus the dimensionless γ_{10}/η and γ_ϕ/η in the case of $\alpha = \beta = 1/\sqrt{2}$, where we have set $\Gamma_1 = \gamma_{10}$, and $t_D = \pi/2\eta$.

In the following, we investigate how to realize the NVE-based quantum memory using RI or DI. The Hamiltonian for this hybrid system is given in units of $\hbar = 1$ by,

$$\begin{aligned} H_{tot} &= \frac{\omega_{10}}{2}\sigma^z + \omega_c a^\dagger a + \frac{\omega_{eg}}{2}S^z \\ &+ g_{tc}(\sigma^+ a + \sigma^- a^\dagger) + g_{td}(S^+ a + S^- a^\dagger). \end{aligned} \quad (2)$$

In the RI case, as shown in Fig. 2(a), we should tune the frequency of the CBJJ to be resonant with the TLR and NVE, namely, $\omega_{10} = \omega_c = \omega_{eg}$, and for simplicity we assume $g_{tc} = g_{td} = g_0$. In the frame rotating with the TLR frequency ω_c , Eq. (2) becomes $H_R = g_0[(\sigma^+ a + \sigma^- a^\dagger) + (S^+ a + S^- a^\dagger)]$, by which an arbitrary state of

CBJJ $|\Psi_C\rangle = \alpha|0\rangle_C + \beta|1\rangle_C$ is first converted into a superposition state of the microwave photon in the TLR, and then transferred to the NVE as $|\Psi_D\rangle = \alpha|0\rangle_D + \beta|1\rangle_D$ with α and β the normalized complex numbers. Once the single microwave photon is absorbed by the NVE, our goal is achieved, where the total operation time is $t_R = (2k+1)\pi/\sqrt{2}g_0$ with k natural numbers.

In the DI case, as shown in Fig. 2(b), the frequency of TLR should be changed to be detuned from the zero-field splitting of the NV centers by $\Delta_{td} = \omega_{eg} - \omega_c \gg g_{td}$. Additionally, using fast control of bias current I_b and junction capacitance C_J of the CBJJ, we may tune the frequency ω_{10} from being resonant with TLR to a large-detuning case $\Delta_{tc} = \omega_{10} - \omega_c \gg g_{tc}$. As a result, both CBJJ and NVE can dispersively interact with the resonator. In this limit, using the Fröhlich's transformation [18], H_{tot} (Eq. (2)) is rewritten as

$$H_{D1} = \omega_c a^\dagger a + \frac{\omega_{10}}{2} \sigma^z + \frac{\omega_{eg}}{2} S^z + \frac{g_{tc}^2}{\Delta_{tc}} (\sigma^+ \sigma^- + \sigma^z a^\dagger a) + \frac{g_{td}^2}{\Delta_{td}} (S^+ S^- + S^z a^\dagger a) + \eta (S^+ \sigma^- + S^- \sigma^+), \quad (3)$$

where $\eta = g_{tc}g_{td}(1/2\Delta_{tc} + 1/2\Delta_{td})$ is the effective CBJJ-NVE coupling rate. If the TLR is initially prepared in the vacuum state $|0\rangle_T$ then the Hamiltonian H_{D1} becomes

$$H_{D2} = \left(\frac{\omega_{10}}{2} + \frac{g_{tc}^2}{2\Delta_{tc}}\right) \sigma^z + \left(\frac{\omega_{eg}}{2} + \frac{g_{td}^2}{2\Delta_{td}}\right) S^z + \eta (S^+ \sigma^- + S^- \sigma^+). \quad (4)$$

Assuming $\omega_{10} = \omega_{eg}$ and $g_{tc}^2/\Delta_{tc} = g_{td}^2/\Delta_{td}$, we have the effective Hamiltonian in the interaction picture as $H_D = \eta(S^+ \sigma^- + S^- \sigma^+)$, which implies that the photon-assisted transitions cannot happen in practice, but the CBJJ-NVE interaction is effectively induced. Different from the RI case, the DI case is usually mediated by the exchange of virtual photons rather than real photons, which could effectively avoid the TLR-induced loss. Therefore the DI method does not require exact time control for the coupling of the photon qubits to the CBJJ and to the NVE. In this way, we can also achieve our goal of NVE-based quantum memory, i.e., $(\alpha|0\rangle_C + \beta|1\rangle_C)|0\rangle_D \rightarrow |0\rangle_C(\alpha|0\rangle_D + \beta|1\rangle_D)$, where the degree of freedom of the TLR mode has been eliminated, and the total operation time is $t_D = (2k+1)\pi/2\eta$.

Taking account of the decoherence effects, we simulate the dynamics of the transfer process by integrating the full phenomenological quantum master equation

$$\dot{\rho} = -i[H_R, \rho] + \frac{\kappa}{2} D[a]\rho + \frac{\gamma_{10} + \Gamma_1}{2} D[\sigma^-]\rho + \frac{\gamma_\phi}{2} D[\sigma_z]\rho, \quad (5)$$

where $D[A]\rho = 2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A$, and κ is the TLR decay rate. γ_{10} and Γ_1 are the spontaneous emission rate and quantum tunneling rate of the level $|1\rangle_C$ of the CBJJ, respectively, and γ_ϕ is the pure dephasing rate of the CBJJ. We characterize the transfer process for some given initial states of the CBJJ by the conditional fidelity

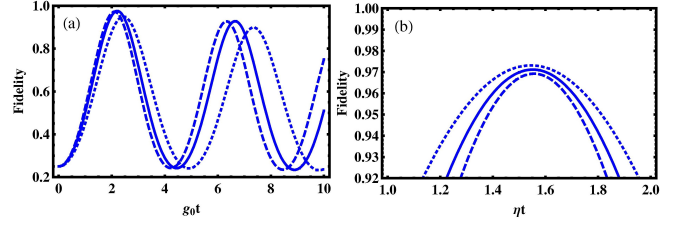


FIG. 3: (Color online) (a) The fidelity of the quantum state transfer versus the dimensionless time $g_0 t$ with $\delta = 0$ (solid), -0.1 (dotted), and 0.1 (dashed) in the case of $\alpha = \beta = 1/\sqrt{2}$, $\gamma_\phi = \Gamma_1 = \gamma_{10} = \kappa = 0.01g_0$. (b) The fidelity of the quantum state transfer versus the dimensionless time ηt with $\alpha = \sqrt{1/2}$ (solid), $\alpha = \sqrt{1/3}$ (dashed), and $\alpha = \sqrt{2/3}$ (dotted), where $\gamma_\phi = \Gamma_1 = 0.015\eta$, and $\gamma_{10} = 0.03\eta$.

of the quantum state according to the expression $F = \langle \Psi_D | \rho(t) | \Psi_D \rangle$, where $|\Psi_D\rangle$ is the target state $\alpha|0\rangle_D + \beta|1\rangle_D$ to be stored in the NVE. In Fig. 2(c), we plot the fidelity F as a function of the TLR decay rate κ and CBJJ decay rate γ in the RI case. One can find that high fidelity could be achieved in the weak decoherence case, and the influence from CBJJ seems more serious than that from TLR. In the DI case, the TLR-induced loss could be effectively suppressed due to the large-detuning. As shown in Fig. 2(d), we can also obtain high fidelity if the CBJJ decay rate γ/η is within the region $[0, 0.01]$.

In the RI case, we assume $g_{tc} = g_0$ and introduce the parameter $\delta = (g_{td} - g_{tc})/g_0$. In real experiments, δ is inevitably non-zero, which leads to errors. But this imperfection is nearly negligible (See Fig. 3(a)) if δ is within the region $[-0.1, 0.1]$. Moreover, the state transmission process accelerates for $\delta > 0$, but decelerates in the case of $\delta < 0$. In the DI case, the fidelity F is plotted in Fig. 3(b) against the operating time for different initial states of the CBJJ, where the nearly perfect state transfer with the fidelity close to 0.97 is available if the parameters are chosen as above, and the fidelity maximum is nearly unchanged with different initial states of the CBJJ. Noted that the decoherence effects resulted from the imperfect spin polarization of the NVE could also reduce the fidelity. Nevertheless, experimental progress has demonstrated the possibility to suppress the spin depolarization by using sufficiently long optical pulse [19].

We survey the relevant experimental parameters. The two methods above require different conditions in implementation. For example, in the RI case, the TLR with the inductance $F_t = 60.7$ nH and the capacitance $C_t = 2$ pF leads to a full wave frequency $\omega_c/2\pi = 2.87$ GHz. The CBJJ's parameters should be tuned to $C_J = 71.5$ pF, $C_c = 60$ fF, $I_c = 67$ μ A, and $I_b/I_c \approx 0.99$, which make $\omega_{10} = \omega_c$ and yield the CBJJ-TLR coupling rate $g_{tc}/2\pi = 10$ MHz. In the DI case, we may choose $\omega_c/2\pi = 2.62$ GHz and the detuning from the CBJJ and NVE by $\Delta_{tc}/2\pi = \Delta_{td}/2\pi = 250$ MHz. As a result, in order to obtain a considerable CBJJ-NVE coupling rate $\eta/2\pi = g_{tc}g_{td}(1/2\Delta_{tc} + 1/2\Delta_{td}) = 10$ MHz in the

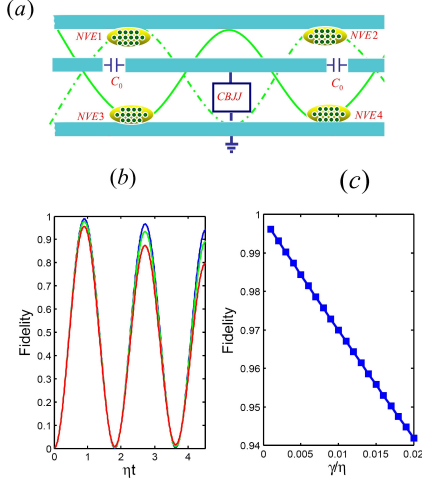


FIG. 4: (Color online) (a) Schematic of the system consisting of many NVEs coupled to the TLR, where the CBJJ acts as a tunable coupler. (b) The fidelity of the state $|W\rangle_3$ versus ηt , where the blue (upper), green (middle) and red (bottom) curves correspond to $\gamma = \eta/50$, $\eta/100$ and $\eta/200$, respectively, and we have set $\gamma = \gamma_\phi = \gamma_{10} = \Gamma_1$. (c) The fidelity of the state $|W\rangle_3$ versus the dimensionless parameter γ/η , where the gating time is $\pi/2\sqrt{3}\eta$, and $\gamma = \gamma_\phi = \gamma_{10} = \Gamma_1$.

case of $g_{tc}/2\pi = g_{td}/2\pi = 50$ MHz, we have to tune the CBJJ's parameters to $C_J = 2.3$ pF, $I_c = 2.177\mu\text{A}$, and $I_b/I_c \approx 0.99$. The dissipation parameters γ_{10} , γ_ϕ , Γ , and κ are on the order of hundreds of kHz [11, 12, 20]. Therefore, both g and η are two orders higher than the dissipation rates, which makes reliable quantum memory feasible. Finally, let us go back to the two-level approximation we have made for the CBJJ-qubit. The leakage probability of quantum state population from the subspace $\{|0\rangle_C, |1\rangle_C\}$ to $|2\rangle_C$ can be estimated as

$P \sim O[g_{tc}^2/(g_{tc}^2 + \Xi^2)] \sim 10^{-3}$ with the level separation $\Xi = |\omega_{21} - \omega_{10}| = \omega_{10}/10$. The small value of P ensures that the two-level approximation in our scheme is reasonable.

One of the favorable applications of our scheme is to entangle many NVEs by coupling to a common CBJJ, which actually acts as a tunable coupler to mediate the virtual excitation of photons. As shown in Fig. 4(a), the effective Hamiltonian of this system in the DI regime reads $H_M = \sum_{i=1}^N \eta_i (S_i^+ \sigma^- + S_i^- \sigma^+)$, where $\eta_i = g_{tc}^i g_{td}^i (1/2\Delta_{tc}^i + 1/2\Delta_{td}^i)$, and N is the number of NVEs. For simplicity, we assume that each NVE is equally coupled to the CBJJ, i.e., $\eta_i = \eta$. So the state evolution under the Hamiltonian H_M is given by $|\Psi(t)\rangle = \cos(\sqrt{N}\eta t) |1\rangle_1 |1\rangle_2 \cdots |1\rangle_N |0\rangle_c + (1/\sqrt{N}) \sin(\sqrt{N}\eta t) \sum_{j=1}^N |1\rangle_1 \cdots |0\rangle_j \cdots |1\rangle_N |1\rangle_c$. By choosing $\sqrt{N}\eta t = \pi/2$, with the readout result of the CBJJ state to be $|1\rangle_c$, we obtain the $|W\rangle_N$ state of NVEs [21] if the system is initially prepared in the state $|\Psi(0)\rangle = |1\rangle_1 |1\rangle_2 \cdots |1\rangle_N |0\rangle_c$ by optically pumping the electron spins in NVEs. The readout of the phase qubit can be accomplished using single-shot measurements [22]. Figs. 4(b) and 4(c) show the fidelity of the state $|W\rangle_3$ under the consideration of decoherence by the quantum master equation $\dot{\rho} = -i[H_M, \rho] + \frac{\gamma_{10} + \Gamma_1}{2} D[\sigma^-] \rho + \frac{\gamma_\phi}{2} D[\sigma_z] \rho$.

In summary, we have put forward a realization of a spin-based quantum memory for CBJJ with currently available techniques. Modulating the external parameters of the CBJJ, we have achieved the quantum information transfer between CBJJ and NVE using RI method or DI method under realistic situation. We argue that our proposal has immediate practical applications in quantum memory and hybrid quantum device with currently available techniques.

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